A Logic of Systems

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The Structure of Wholes

The problem of the integration of part processes in the total organism is the most important and at the same time the most difficult problem for a science of personality. The difficulty lies not alone in the paucity of usable factual data, but to an even greater extent in the inadequacy of our logical tools. Such a handicap is felt not only in the study of personality, but in the study of wholes in general. An attempt will be made to develop some concepts which may be useful for the understanding of the structure of wholes.

Our scientific thinking prevalently in the logical manipulation of relationships. That the structure of wholes cannot be described in terms of relationships has, however, been repeatedly pointed out by many writers. While accepting the premise that holistic connexions cannot be resolved into relationships, some authors have implied that the pattern or structure of wholes does not lend itself at all to logical manipulation. We suggest, however, that the structure of wholes is perhaps amenable to logical treatment after all, that, though it may not be described in terms of relations, it may be described in terms of some more adequate logical unit, representing an entirely different logical genus. Here the attempt will be made to demonstrate that there is a logical genus suitable to the treatment of wholes. We propose to call it *system*.

The ideal would be to develop a logic system to such a degree of precision that it might offer the basis for exact mathematical formulation of holistic connexions. A. Meyer states, "The mathematics which would be needed for the mathematical formulation of biological laws does not exist today. It has been created by the new biology" (Meyer, 1934, p. 35). To construct a logic of system which would be the counterpart of the conventional logic of relations is in itself a gigantic task and cannot even be attempted here. It may be true that a substantial advance in the study of wholes, and specifically in the study of personality, will come from the (note **Figure 1)**
development of a logic systems. We must, however, content ourselves for the present with the clarification of some aspects of the logical properties of systems and with the application of the insight thus gained to our specific subject-matter. (note **Figure 2)**

In order to demonstrate some of the logical characteristics of systems we may compare them with better known logical forms, namely, with relationships. As an example of a relationship we may take a quantitative one (Figure 1), and as an example of a system we may take a simple geometrical one (Figure 2) in which the points a, b, c, and d are parts of the simple geometrical system, the line A-B.

The differences between relationships and systems may be formulated as follows:

1. A relationship requires two and only two members (relata) between which the relationship is established. A complex relation can always be analysed into pairs of relata, while the system cannot be thus analysed. A system may involve an unspecified number of members. A system is not a complex relation. It is impossible to say what the relation between a and b, b and c, and c and d, etc., should be in order for form a linear system.

I am aware of the fact that the restriction of the term >relation= to cover only two-term connexions deviates from the contemporary usage of this term. Usually the term is employed also to include logical connexions that involve more than two terms. Such connexions, however, seem to fall into the following two categories:

(a) Compound relations, which can be reduced to two-term relations. One or both of the relata may be groups involving a more or less large number of members. Group A may include a, b, c, and d, and group B may include e, f, g, and h. When, however, group A is related to B, or to one member of B, the group is taken as a totality, that is, as *one* unit. A compound relation may involve also a chain connexion, for instance, a >casual= sequence: a-b-c-d. It is clear that a compound relation can easily be resolved into two-term relations: a-b, b-c, c-d. A more complex relationship would be as in Figure 3, which consists of the two-term relations: a-b, b-c, b-d, d-e, c-e. The complexity may be even greater and yet a reduction of two-term relations still is possible.

(b) There are also connexions which involve more than two terms and cannot be reduced to two-term relations as, for instance, b is between a and c. It appears, however, that those compound connexions which cannot be reduced to two-term relations exhibit all those qualities which are set forth in this discussion as the characteristics of systems. One may, of course, use the term >relations= in a very broad sense as is commonly done, but then one must admit that one subsumes two very different logical genera (two-term relationships and complex connexions which are reducible to two-term relations on the one hand, and complex connexions which cannot be reduced to two-term relations on the other hand) under the same term.

The term >system=, as used in this discussion, is also at variance which the common usage. Usually one designates by system any aggregate of elements considered together with the relationships holding among them. It will be shown in the following discussion that the type of connexions in a whole is very different from the connexions which exist in an aggregate. The term >system= is used here to denote a *holistic system.* Further, in using this term we abstract *constituents* (>elements=) and refer only to the *organization* of the whole. Thus >system= for our discussion is holistic organization.

It might seem desirable in the present discussion to substitute other terms for >relation= and >system=; however, it seems to me equally desirable to avoid the coining of terms. Since I have pointed out the differences between my usage and the common usage of the terms I may hope that the argument will not become obscured through this terminology.

2. A relation requires an aspect out of which the relationship is formed. Two objects can be related to each other, for instance, with regard to their color, size, or weight. Therefore, before a relationship can be established it is necessary to single out some aspect of the relata which serves as a basis of the relation. The attribute of the relata on which the relationship is based is an immanent quality of the object, like size, color, or weight. The object enters into a relationship with another object because of its immanent qualities. Most relationships are based upon >identity=, diversity, or similarity (partial identity with partial diversity) of the object, that is, on immanent attributes. The members of a system, on the contrary, do not become constituents of the system by means of their immanent qualities, but by means of their distribution or arrangement within the system. The object does not participate in the system by an inherent quality but by its *positional value in the system.* It is immaterial for a linear system whether points or starts or crosses or circles or any other objects be the members, if only in the arrangement the positional values remain the same.

Between the constituents of a system, after they gain a positional value from the system, further relations may be established in a *secondary way* which are not based on the immanent properties of the relata, but on their secondary positional value. Such relationship is, for instance, A is below B. Such relations are secondary: it is presupposed that the members have a positional value in a system of coordinates.

3. In establishing a relationship between objects and in arranging objects in a system, the separation of the objects is presupposed. Multiplicity of objects is only possible in some kind of *dimensional domain* (a manifold). The clearest examples of dimensional domains are space and time, which have been reorganized by philosophers for a long time as *principia individuationis*, that is, domain which make possible a multiplicity of individual objects. We cannot speak of *two* objects unless they are placed in different points of time or in different points of space or unless a distance between them is not established in some other kind of dimensional domain.

Although the dimensional domain is a necessary condition for both relationships and systems, the function of the dimensional domain is different for these two logical genera. The role of the dimensional domain for a relationship is merely disjunction of the relata. But the role of the dimensional domain ends here. The domain itself does not enter into the relationship. For instance, if two colors are separated in space a comparison between them can be made without any further reference to space.

The dimensional domain is more intimately involved in the formation of systems. Here the dimensional domain not only separates the parts, but it participates in the formation of the system. The system is dimensional. *A system is a distribution of the members in a dimensional domain.*

4. In a relationship the connectedness between the relata is a *direct* one. The connexion goes without any mediation directly from a to b and vice versa. The connexion between the members of a system is, however, a more complex type. Although there is a connexion between the points a, b, c, d when they form a straight line, this connexion is not a direct one in our sense. It is impossible to say what relationship should connect a with b, and c with d and a with d, etc., to form a linear arrangement. In this example the members of the system which are points are linearly connected only by forming a whole. System-connectedness of the parts cannot be expressed as a-b, b-c, a-c, but as in Figure 4.

In a system the members are, from the holistic viewpoint, not significantly connected with each other except with reference to the whole.

The constituent parts of a system are not considered separately but with respect to a superordinate, more inclusive factor, the (note **Figure 5**) system in and by which they are connected. An interesting example of this state of affairs is given in the fact of geometrical symmetry. Such an arrangement involves two figures which are identical in shape and size, and shows a special kind of correspondence with regard to their positions in space. The identity of geometrical figures can be demonstrated by bringing them together in space, in which case they must coincide. Symmetrical figures, however, can coincide only under special conditions. Taking one-dimensional geometrical figures such as those in Figure 5, it is clear that such figures cannot be made to coincide by moving them in a one-dimensional space, that is, along the line a-b, A_1 will not coincide with A_2 and B_1 and B_2 at the same (note **Figure 6)** time. To make them actually coincide they must be rotated within a two-dimensional space with respect to the axis of symmetry. Thus the position of the two lines is analogous, not in a one- but in a two-dimensional space. Taking two-dimensional symmetrical figures such as those in Figure 6, it can easily be seen that they cannot be made to coincide in any way in the concrete if one shifts them within a two-dimensional space, but only within a three-dimensional space. Three-dimensional symmetrical objects again cannot be made to coincide in a three-dimensional space. That would require in the concrete a fourth dimension. If one=s mirror image were real and one could go beyond the mirror, one still could not step into one=s mirror image and completely coincide with it, because the right side of the person would be on the left side of the mirror image and vice versa; a right-hand glove cannot be made into a left-hand glove; and so on. For the coincidence of symmetrical figures, a space which has one dimension more than the number of dimensions of the symmetrical figures is required. Thus the congruence of one-dimensional space, and that of two-dimensional figures in a three-dimensional space. Symmetry seems to be psychologically a kind of system-connectedness, that is, a connectedness of parts, not between themselves but in a superordinate whole.

Up to this point we have considered only one type of relations, namely, comparative relations based on >identity=, diversity, and similarity. Another type of relationship which is of great significance in science is the casual directly connected; they are connected within the and by the whole.

Dealing with relations and dealing with systems involve two different logical manipulations to which two psychologically quite different processes may correspond. In the recent past there has been much rather inconclusive discussion concerning the possibility of two different processes of knowing: explanation and understanding. I am referring here to discussion of the problem, *erklärende und verstehende Psychologie.* The difference between the two concepts, as they have been used in the aforementioned discussion, is probably that explanation refers to relational thinking, understanding to system thinking. Relational thinking aims at the establishment of the direct connexion between two objects. For instance, in the study of causation one has to find for member a (effect) a second member b (cause) with which it is necessarily connected. In casual research the task is to single out from a multiplicity of data pairs of facts between which there is a necessary connexion. In system thinking the task is not to find direct relations between members but to find the superordinate system in which they are connected or to define the positional value of members relative to the superordinate system.

The preceding discussion is far from being complete and adequate, but it is hoped that our main thesis at least, that relations and systems are two different logical genera with distinctive characteristics, has been given a relationship. Casual relationship can be expressed as follows: if a occurs, b follows. Casual relationships may be in many respects different from comparative relationships, but they are alike in that they represent direct connexions between one member and another. Even complex casual relationships can be analysed into two-member relationships. Where such analysis is logically not possible, we deal with system-connectedness.

On the basis of the foregoing discussion the differences between relations and systems can be summarized in a preliminary fashion as follows: (1) Relationships involve two and only two members (relata). Complex relationships can always be analysed into pairs of relata. Systems may involve an unspecified number of components, not analysable in certain respects into pairs of relata. (2) The relata enter into a relationship by virtue of their immanent attributes, while the constituents enter into a system-connexion, not through their immanent attributes but through the positional value which they have in the system. But the system itself cannot be described even in terms of such relationships. (3) For the existence of systems a dimensional domain is necessary. Systems are specific forms of the distribution of members in a dimensional domain. (4) In relationships the connexion between the relata is a direct one. The members of a system, on the contrary, do not need to be measures of plausibility. It is still an open question whether relations and system are *absolutely* different logical genera, or whether the one may be a subtype of the other. The latter possibility cannot be excluded and there are certain arguments in favor of it. One thing, however, seems clear, namely, that systems cannot be deducted from relations, while the deduction of relations from systems still remains a possibility. If that is the case then the more general logical genus would be >system=, while >relation= would be a reduced, simplified system which is adequate only for the logical presentation of very simple specialized constellations.

System and Gestalt

In the course of the past two decades it has been almost generally recognized by biologists and psychologists that the clarification of the problem of wholes is essential for progress in the study of the organism. The increasing awareness of the problems of wholes led to the discovery of certain general principles, best formulated perhaps by the Gestalt psychologists. It will be useful to examine briefly these formulations in the light of the previous discussion.

The most generally known thesis with regard to wholes is the following: >The whole is more than the sum of its parts.= This is not a very felicitous formulation since - contrary to the concept of Gestalt psychologists - it may suggest that a summation of parts takes place and that, besides the summation, a new additional factor enters into the constitution of wholes. In Ehrenfels= *Gestaltqualität* such an additional factor actually has been suggested.

Wholes, however, cannot be compared to additive aggregations at al. Instead of stating that in the formation of wholes something more than a summation of parts takes place, it would be more correct to state that summation does not play any part whatsoever in the formation of wholes. In summations the parts function because of their inherent qualities. When, for instance linear distances are added to form a larger linear distance, the first distance, as such, is directly joined (*und-Verbindung*) to the second distance and this to the third, and so on. On the other hand, when a number of parts constitute a whole, the parts do not enter into such a connexion by means of their inherent qualities, but by means of their position in the system. The formation of wholes is therefore not additional to the aggregation of parts, but something of an entirely different order. *In aggregates it is significant that the parts are added; in a system it is significant that the parts are arranged.*

It should also be kept in mind that >part= means something different when applied to aggregate from what it means when applied to wholes. When the single objects a, b, c, d, are bound together in an aggregate they participate in that aggregation as object a, object b, object c, etc., that is, as lines, distances, color spots, or whatever they may be. When, however, a whole is constituted by the utilization of objects a, b, c, d, the parts of the resulting whole are *not* object a, object b, object c, etc., but α, β, γ, δ, that is, *the positional values of the objects* a, b, c, d.

I would suggest that the principle >the whole is more than the sum of its parts= be modified in the following way: aggregation and whole formation are processes of an entirely different order. And we may also formulate this statement more specifically: in an aggregation the parts are added, in wholes the parts are arranged in a system. The system cannot be derived from the parts; the system is an independent framework in which the parts are placed.

That the whole is, to a large extent, independent of the individual parts has been frequently pointed out. We may transpose a melody a few octaves higher or lower and it still remains essentially the same melody, although this transposition may be such that the two variations of the melody have no single individual tone in common. If we recall that this system is a kind of arrangement in which the parts do not participate by means of their inherent characteristics but by means of the positional values, the above-mentioned relative independence of the whole from the nature of the individual parts will be understandable.

The above statement needs, however, some qualification and restriction. The parts may need to have certain attributes which enable them to fill the positions which are required of the system. In a triangular geometrical arrangement the parts have to be *lines*, although their other properties (for instance, their absolute lengths) are irrelevant. Thus certain properties of the constituents are relevant, that is, they are necessary to permit the occupancy of a given position, while other properties are irrelevant. A similar distinction has been made also by J. von Uexküll, who differentiates between >leading= and >accompanying= properties (Uexküll, 1928). The greater the organization of the whole, the more the inherent properties of parts are utilized as co-determinants of positional values. The human organism, for example, is highly economical in this respect: it carries a minimal load of irrelevant properties; most of the properties of parts are >utilized=, that is, are co-determinants of the positional value of the part.

>Wholes are never undifferentiated but always a *unitas multiplex.*= Let us place the emphasis first on the multiplicity. If one keeps in mind that the system is a way of arranging parts, the logical necessity of a multiplicity becomes evident since a single factor in itself cannot be arranged.

The term >whole= is frequently used with a very confusing double meaning. Sometimes the concrete organized *object*, other times the *organization* of the object is called a whole. The term is used in this latter connotation, for instance, when one states that a circle may be small or large, drawn in red or in green color, and still remain in the same whole, the same Gestalt. I propose that the term >whole= be reserved to designate the concrete *organized object,* while the *organization* itself, the way of arrangement of parts, should be called system.

The logical formulation of a given system states the construction principle or the *system principle* of the whole. Every system has one and only one construction principle. This is the meaning of the first term in the expression, *unitas multiplex.* The particular system principle may be perfectly or only approximately realized in a given whole. There are wholes in which all the significant positions of the system are occupied in perfect accordance with the system principle, but there are also wholes in which only a limited number of positions, sufficient to suggest the system principle, are occupied while other members are *out of position.* This is the difference which among Gestalt psychologists is somewhat vaguely referred to by the terms >good= and >bad= Gestalt. The various degrees of *Pregnanz* which a Gestalt may have express the degree of conformity of the positions of the parts with the system principle. There are also instances where in a whole a sufficient number of positions are occupied to indicate the system principle, while the other positions *are not filled.* These are >open= Gestalts in contradistinction with >closed= ones, wherein all significant positions are occupied.

Certain relations, as, for instance, comparative relations (a is larger than b), could be called static, while others could be called dynamic. The prototype of the latter is the casual relationship. In the same fashion one could distinguish between static and dynamic systems. We hope we have demonstrated the existence of the static forms of systems. The question, however, as to the existence of dynamic systems still remains open. In static systems the whole imparts to its constituents a positional value which the given constituent does not have in itself, but only when it forms part of the given whole. With regard to dynamic wholes, one would expect that a given part *functions differently* depending on the whole to which it belongs. We would also expect that the whole has its own characteristic dynamics. Certain principles of holistic dynamics have already been formulated, as, for instance, the >tendency to closure= and the >tendency to *Pregnanz*=. Up to the present such dynamic principles have been satisfactorily demonstrated only within the psychological realm, which is generally characterized by great plasticity. That such principles hold true also in realms of greater rigidity is strongly suggested by certain facts, but it has not been satisfactorily demonstrated as yet. Such an assumption is a working hypothesis - and in the present discussion it is taken as such; a convincing demonstration remains a problem for future inquiry.

The possibility of the dynamic action of a system would probably be rejected *a priori* by many students. Although in the last analysis causality is just as inexplicable as a system action, still many students would feel more comfortable and would be willing to give credit for greater scientific validity to the formulation of the dynamics of a given happening in terms of causality than to its formulation in terms of system action. Casual thinking has been used in science for such a long time and, in certain fields, with such success that it is almost generally considered as *the* scientific thinking, although it may well be only a subvariety of it. Relational thinking is at least as difficult as the transition from a three-dimensional to a four-dimensional geometry.

This brief discussion of some holistic principles and concepts suggests the possibility of the logical formulation of some principles and concepts. Only a strict logical formulation can dispel the vagueness and obscurity which have been so common in the early holistic theories.
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References
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UEXKÜLL, J. VON (1928), *Theoretische Biologie*, Springer, 2nd edn.

Figures 1 through 6

Figure 1:

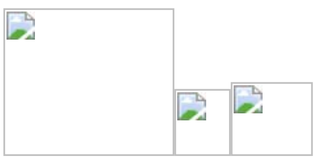

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Figure 2:

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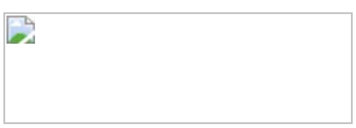
Figure 3:

c



a b d e

Figure 4:

A

A B C D

Figure 5:

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D_1 C_1 C_2 D_2

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